Clifford Algebra and GIS Spatial Analysis Algorithms – the Case Study of Geographical Network and Voronoi Analysis

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ABSTRACT

Introducing Clifford Algebra as the mathematical foundation, we proposed a unified multi-dimensional GIS data model, constructed by linking data objects of different dimensions within the multivector structure of Clifford algebra. Then, algorithms for geographical network analysis (such as shortest path, minimum unicom and maximum flow analysis) and high-dimensional voronoi diagram were constructed. We use both simulation and real world data to test the usability and performance of our algorithms. The result gives very positive prospect of implement GIS analysis algorithms under Clifford Algebra framework. In that way, traditional GIS analyses algorithms can be extended not only accommodate various dimensions but also get beneficial on performance.

Keywords

Clifford Algebra, GIS, Multi-Dimensional, Algorithm.

1. INTRODUCTION

Clifford algebra, which brings scalar and vector algebra into a unified framework and is widely used in various fields, brings us powerful tools to express multi-dimensional objects, extending classical geometric analysis and inherit and transplant existing analysis methods under unified framework. In this article, we introduce Clifford Algebra in GIS, and propose a unified multi-dimensional data model and several practical GIS analysis algorithms, which aim to evaluate the usability and efficacy of introducing Clifford algebra into multi-dimensional GIS spatial analysis. We also discussed their tests on both real world geographical and simulated data.

2. DATA MODEL

"Support data models for a complete range of geographic phenomena" and "support a wide range of types of geographic simulation" have already been identified as two of the 10 "grand challenges" for GIS [Lon05]. GIS data models should be developed for modeling complex geographic phenomena, whereas current GIS systems show limitations in representing real geographic phenomena [Yua09]. Some traditional GIS data models, separate objects of different dimensions and processing them separately, which increases the complexity of modeling and expressing real world data and splits and blurs the spatio-temporal semantics. Other models, like the event-based [Peu95] or the object-oriented data models [Wac94], also suffer dimensionality problems and compatibility with existing GIS data. Recently developed data models extend some of the above-mentioned aspects, but still meet a lot of difficulties in seamlessly integrating GIS data of different dimensions [Liu08].

We attempt to unify the multi-dimensional GIS data by associating the objects of different dimensions. The unified data model can be universally described as (x,y,z,a), where "a" means attributes that should be include in analysis process (Figure1). We can reorganize multidimensional GIS data by the combination of different dimensions, from two dimensions (x-a, y-a, z-a, etc.), three dimensions (x-y-a, x-z-a) to four dimensions (x-y-z-a). In order to analyze the characteristics, relationships and relative processes with Clifford algebra, we need to transform the geographical space into Clifford space. At last, based on the multi-dimensionality of the blades, we can construct corresponding data classes for the description of data objects with different dimensions with in a single multivector data structure, which can be further analyzed with various geometric operators.

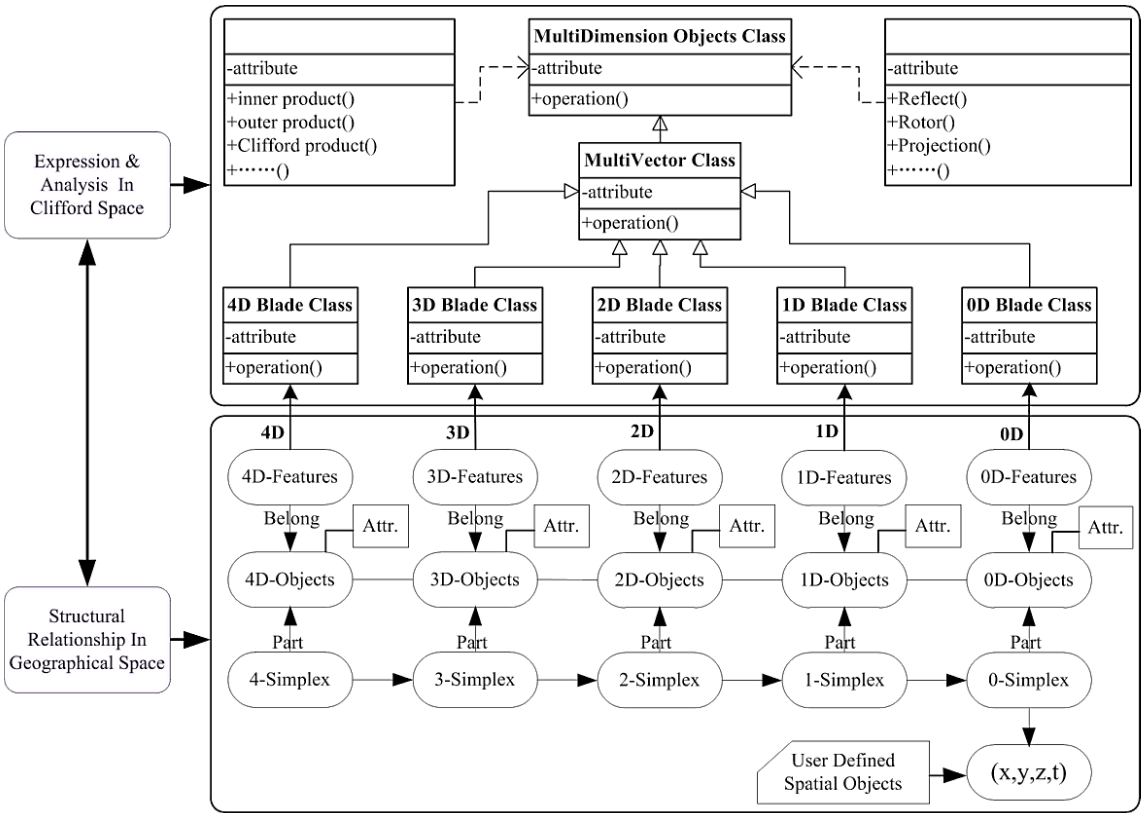


Figure 1. The structure of unfiled spatial-temporal data model

3. GEO-NETWORK ANALYSIS

Network analysis remains one of the most significant and persistent research and application areas in GIS. Many network analyses are among the most difficult issues to solve in terms of their combinatorial complexity [Cur07]. Some early researches on graph theory have proved that Clifford algebra can express the dynamic evolution of a directional and unidirectional finite graph, and can greatly reduce the computational complexity (e.g. [Sch10]). The geometric characteristics (direction, distance, topology and adjacent relationships) can be easily modeled by coding the nodes and routes with Clifford elements. We propose Geographical Network Analysis algorithms that can automatically construct the shortest path, minimum unicom and maximum flow from raw geographic networks without complex preprocessing. The flow chart of each algorithm was shown in Figure2. The above algorithms are implemented in C++ as a module of Unified Temporal-Spatial Analysis System based on Clifford algebra (CAUSTA) [Yua10].

We use China High-way data to test the correctness of our algorithms. The road network test result was shown in Figure3, comparing them with output of traditional GIS network analysis suggests the output of our algorithms and traditional GIS network algorithms are the same, which suggest our algorithms are correct. The spatial and temporal efficiency of our algorithms are computed using random network with different nodes and paths. Taken shortest path for example, the numbers of nodes varied from 1,000 to 10,000, and each network was repeated 10 times with random start and end nodes. Avoiding the use of the adjacent matrix simplifies the design and extension of the algorithm, so that our algorithm has much less memory usage, and can be extended for large scale network analysis (Table1). We also have two experiment with 50,000 and 100,000 nodes networks to test our algorithm capability of large scale data. Even with this large numbers of nodes, the 10 times means of time and memory usage are only 59.153s/851.172s and 12333Kb/27471Kb .

N. Nodes	N. Paths	Dijkstra Time Mean(s)	Our Time Mean(s)	Our Time Stdev(s)
1000	2866	0.049	0.043	0.003
3000	5890	0.533	0.123	0.005
5000	10428	1.506	0.345	0.023
8000	15414	3.964	0.671	0.045
10000	30244	6.652	2.298	0.117

Table 1. Test Networks statistics and time performance compared with Dijkstra algorithm

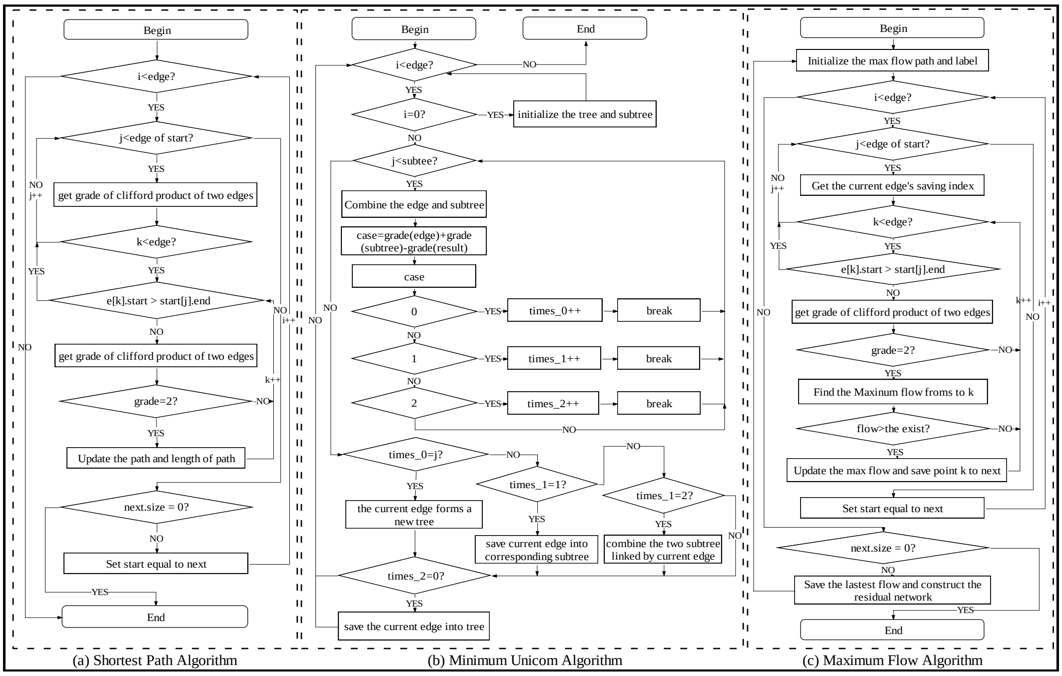
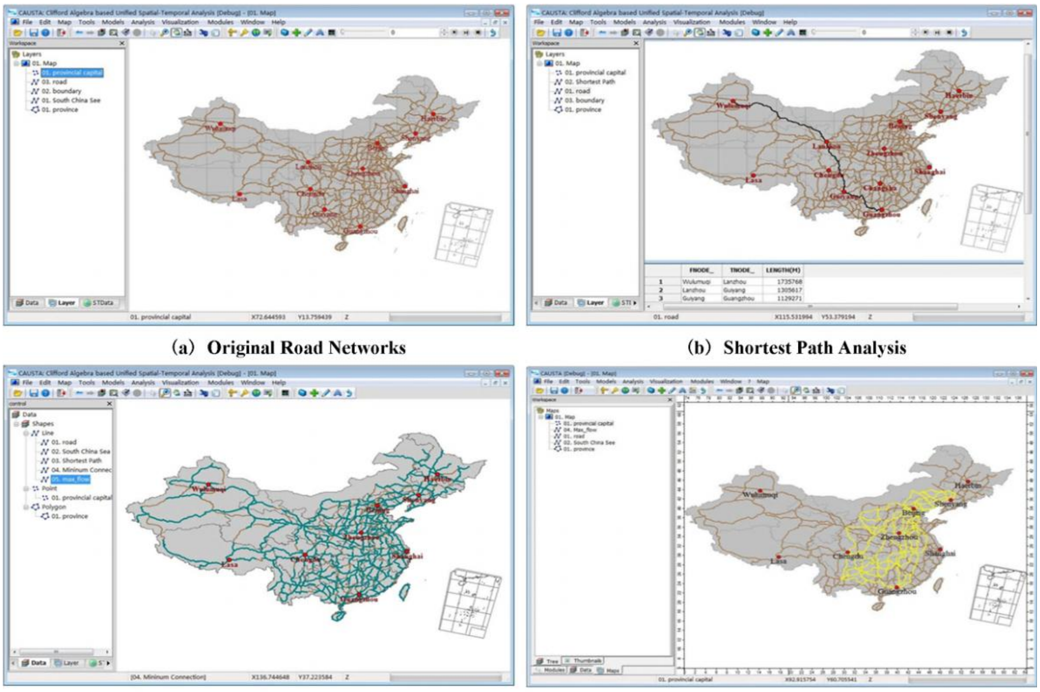


Figure 2. Flow Chart of Network Analysis Algorithms



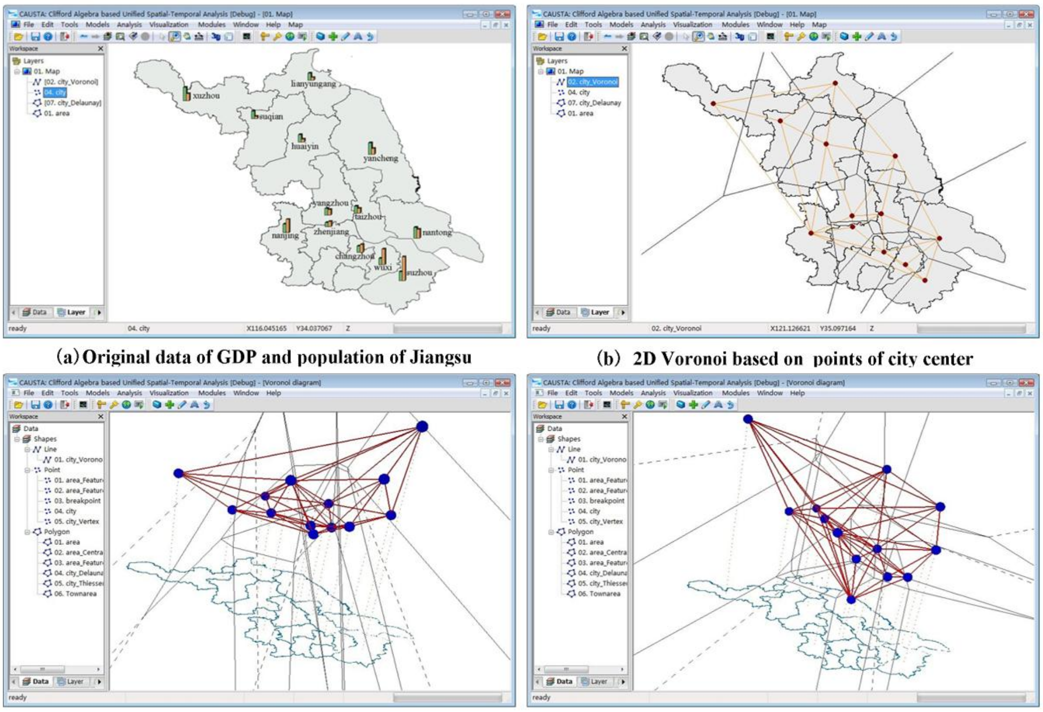
(a) Original Road Networks (b) Shortest Path Analysis

(c) Minimum Unicom Analysis (d) Maximum flow analysis

Figure 3. Network Analysis of China Highway Road Network

4. HIGH-DIM VORONOI DIAGRAM

Delaunay triangulation and Voronoi diagram can be applied in many data organization and analysis tasks (e.g. [Sam06]). Most of their implementations are typically for 2-d objects. For higher dimensional data, performance efficiency, flexibility, and extendibility of data structures are more complex. Several higher dimensional Voronoi analysis algorithms have been proposed under Clifford Algebra framework (e.g, [Zon07], [Dor07]). However, existing algorithms are not very perfectly suited for GIS data. Here, we use [Zon07]'s programming framework but define data structures of our own except using CGAL, which may provide more extensibility for GIS spatial data. The test of our algorithm can be seen in figure 4. See [Yua10] for details of the algorithms characters.



(a) Original data of GDP and population of Jiangsu　　(b) 2D Voronoi based on points of city center

(c) 3D Voronoi （GDP as the third dimension）　　(d) 3D Voronoi （Population as the third dimension）

Figure 4. Voronoi Diagram based on Clifford algebra (Adapted from [Yua10])

5. CONCLUSIONS

In this article, we introduced Clifford algebra into multi-dimensional GIS spatial analysis. We proposed a unified data model that can represent multi-dimensional GIS data as standalone data objects in multivector form. Two kinds of GIS analysis algorithms: the geographical network analysis algorithm and the high dimensional Voronoi diagram were then proposed. Test on different kinds of geographical data suggest both the correctness and performance of our algorithms. All of the above suggest Clifford algebra can be seen as a new, powerful tool for multidimensional GIS analysis.

6. ACKNOWLEDGMENTS

Thanks for support by the National High Technology R&D Program of China (Grant no. 2009AA12Z205) and Key Project of National Natural Science Foundation of China (Grant no. 40730527).

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